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SPHERICAL METHOD FOR MEASUREMENT OF ANGULAR VELOCITY

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ABSTRACT:

The goal of this research has been to develop a measurement system using linear accelerometers to determine the 3-D motion of various dummy components during impact. The spherical procedure was developed to track dummy motion when film documentation was not reasonable, such as during air bag deployment. the procedure uses three or more accelerometer triaxial clusters for assessment of angular velocity and linear acceleration. From these measurements, angular acceleration and position, and linear velocity and position are derived.

INTRODUCTION:

The concept used is different from other procedures which estimate three dimensional motion from linear accelerometer mounted at various locations on a rigid body. The approach used by most other methods is to view the structure undergoing 3-D motion as a rigid body with 3 components of linear acceleration and three components of angular acceleration. These approaches generally find the acceleration gradient across the rigid body and then interpolate this gradient as a combination of angular acceleration and angular velocity. Through various mathematical procedures and digital data manipulation, the angular acceleration is derived and then the angular velocity is obtained by integration. The spherical method is derived from concepts used in the geometric methods currently being applied to fundemental physics problems (1-5). In the spherical method presented here, each triaxial cluster is viewed as a point moving on a sphere. The velocity of each moving point with respect to the center of the sphere is obtained through algebraic multiplication of its specific triaxial cluster acceleration in its local instrument frame. Once the liner velocity of each moving point on the sphere is obtained, the angular velocity and the angular acceleration can be derived algebraicaly.

TWO DIMENSIONS:

To understand the method of finding the three dimensional motion that the spherical procedure uses, it is instructive to first look at the two dimensional case. Assume that there are two orthongonal accelerometer arrays on a flat disk (Figure 1). The linear acceleration is obtained by summing the two accelerations.

$$\frac{A_{X2} - A_{X1}}{2} = A_I \quad \text{and} \quad \frac{A_{Y2} - A_{Y1}}{2} = A_J$$

the tangential acceleration A^t along the disk surface is

$$A^t = A_{x2} - A_I \quad \text{or} \quad A^t = A_I - A_{x1}$$

The tangential velocity (along the disk surface) is not as easily obtained. This is because there are two solutions to the velocity equations:

$$A_{y2} - A_J = \frac{V^2}{R}$$

$$V = \pm\,((A_{y2} - A_J\ ) * R\ )^{1/2}$$

However, it is easy to determine the correct velocity by a short duration integration of the tangential acceleration starting from a previous velocity

$$V_{T1} + \int_{T1}^{T2} A^t\ dt = V_{T2}$$

then choose the result closest to zero:

$$(+V - V_{T2})$$

$$(-V - V_{T2})$$

The angular acceleration is then

$$\mathring{\omega} = \frac{A^t}{R}$$

The angular velocity becomes

$$\omega = \frac{V}{R}$$

If there is noise or error in the system, then in general:

$$\omega \neq \int \mathring{\omega}\ dt$$

Although, in three dimensions, the equations become more complicated, the basic principles are the same.

THREE DIMENSIONS:

The following discussion gives a general overview of how this spherical method procedure calculates angular velocity.

Consider three triaxial accelerometers constrained to move on a sphere (Figure 2), which is free to translate without rotation.The rotational motion of the rigid body is represented by the movement of the triaxial accelerometer clusters on the surface.

Consider that two of the accelerations of each of the triaxial clusters are tangent to the sphere and that the third is normal to the sphere.

Assume (x1,y1,z1), (x2,y2,z2), (x3,y3,z3) are the local coordinate systems used for each of the three triaxes on the sphere. Let(I,J,K) be the global instrument frame, and (x,y,z) be the laboratory frame, then:

$$\begin{pmatrix} x1 \\ y1 \\ z1 \end{pmatrix} = \begin{bmatrix} E_{12} \end{bmatrix} \begin{pmatrix} x2 \\ y2 \\ z2 \end{pmatrix} = \begin{bmatrix} E_{13} \end{bmatrix} \begin{pmatrix} x1 \\ x3 \\ z3 \end{pmatrix}$$

Where the E_{ij}'s are the directional cosines or transformation matrix between local coordinate systems, and

$$\begin{pmatrix} I \\ J \\ K \end{pmatrix} = \begin{bmatrix} D_1 \end{bmatrix} \begin{pmatrix} x1 \\ y1 \\ z1 \end{pmatrix} = \begin{bmatrix} D_2 \end{bmatrix} \begin{pmatrix} x2 \\ x2 \\ z2 \end{pmatrix} = \begin{bmatrix} D_3 \end{bmatrix} \begin{pmatrix} x3 \\ x3 \\ z3 \end{pmatrix}$$

Where the D_i's are the directional cosines or transformation matrix between the local and global coordinate systems.

In the (I,J,K) coordinate system:

$$A_i = \sum_{1}^{3} C_i \, A_{ij}$$

Where the C_i's are the coefficients derived from the relative locations of the 3 triaxes, and the A_{ij} are the ith acceleration of the jth triax in the global instrument frame. A_i is the component acceleration in the (I,J,K) coordinate system.

The acceleration of each triax that is related to motion on the sphere surface in the global instrument frame is:

$$As_{ij} = A_{ij} - A_i$$

Transform the AS_{ij} acceleration into the local coordinate system

$$\begin{pmatrix} AS_{X1} \\ AS_{Y1} \\ AS_{z1} \end{pmatrix} = \begin{bmatrix} & & {}^{-1} \\ & D & \\ & & {}_{1} \end{bmatrix} \begin{pmatrix} AS_{11} \\ AS_{21} \\ AS_{31} \end{pmatrix}$$

In the local instrument frame, V_{Ri} is the resultant velocity of each triax as it moves on the sphere. A_{xi} is the normal acceleration for each triax:

$$A_{xi} = \frac{V_{Ri}^2}{R} \quad \text{and} \quad \frac{dA_{xi} * R}{dt} = 2A^t_{Ri} V_{Ri}$$

Where A^t is the tangential acceleration, and R is the distance from the sphere's center to the local triax. A_{Ri} is the resultant acceleration for the ith thriax.

$$\vec{A}_i \cdot \vec{V}_i = A_i^t V_{Ri}$$

In the local coordinate system, the velocity is only in 2 directions. Then

$$A_{yi} * V_{yi} + A_{zi} * V_{zi} = A^t_i V_{Ri}$$

$$A_{Ri} * V_{Ri} (\cos\theta_i \cos\phi_i + \sin\theta_i \sin\phi_i) = A^t_i V_{Ri}$$

where θ_i and ϕ_i are the angles related to the projections of the acceleration and velocity vectors in the local coordinate system:

$$\cos(\theta_i - \phi_i) = \frac{A_i^t V_{Ri}}{A_{Ri} V_{Ri}}$$

$$\mathrm{Arccos}\left(\frac{A_i^t V_{Ri}}{A_{Ri} V_{Ri}}\right) = \phi_i - \theta_i$$

$$\theta = \frac{A_{Xi}}{A_{Ri}}$$

$$\mathrm{Arccos}\left(\frac{A_i^t V_{Ri}}{A_{Ri} V_{Ri}}\right) + \frac{A_i}{A_{Ri}} = \theta$$

Since the experimental data are not exact and contain error, numerical difficulties can result under the following conditions:

$$A_{Ri} * V_{Ri} \approx 0$$

$$A_i^t * V_{Ri} \approx 0$$

A_{yi} and $A_{zi} \approx 0$

when $A_{ri} * V_{Ri} \approx 0$

An integration routine is used to obtain V_y , V_z

when $A^t V_{ri} \approx 0$ then

$$A_{yi} V_{yi} = - A_{zi} V_{zi}$$

$$\frac{A_{yi}}{A_{zi}} = - \frac{V_{yi}}{V_{zi}}$$

$$\frac{A_{yi}}{A_{zi}} \sqrt{A_{xi} R - V_{zi}^2} = V_{yi}$$

When A_{yi} and A_z are small, then an integration routine is used to obtain V_y and V_z.

The above equations, in general, result in two solutions for V. However, the correct solution can be obtained by comparison of the two solutions to an integration derived velocity, similar to the two-dimensional case.

In the application associated with the spherical method presented so far, no use has been made of the constraints which hold the different triaxes at fixed locations on the sphere. Therefore, a least squares routine is used to minimize error in the various velocities, and to keep each triax at a fixed distance from every other triax.

Once the velocity is obtained, then from

$$\vec{V} = \vec{\omega} \times \vec{r}$$

ω can be derived in the least squares sense:

$$\vec{AS} = \mathring{\omega} \times R + \omega \times V$$

$$\mathring{\omega} \times R = \vec{AS} - \omega \times V$$

Once again, $\mathring{\omega}$ can be derived in the least squares sense.

COMPARISON OF THE PROCEDURES:

One method of determining the usefulness of a new analytical procedure is to compare it to other similar procedures that are already in use.

This procedure was compared, using artificially derived motion, to two other multiple accelerometer techniques. The two other procedures chosen were a least square (3-3-3) routine, and the NHTSA version of the Wayne State University System (3-2-2-2). To do this, four triaxial clusters were artificially created (Figure 3). Rigid body motion in all 6 dimensions were introduced (3 translations and 3 rotations). By appropriate choice of accelerometers, this configuration can be used by any of the three procedures to generate rigid body motion.

NOISE:

All three procedures produce correct motion in terms of angular and linear acceleration, velocity, and position, when there is no noise in the system. When noise is introduced into the acceleration signals, then each of the three procedures produces results that differs from the correct motion. The following types of noise were introduced into each of the accelerometers: Gaussian, low frequency, cross-axis and miscalibration. The noise-produced acceleration error that was never greater than 17%

of the resultant acceleration for each triax. This means that during periods of high acceleration, the transducer time-history is predominantly signal, and during low acceleration, the transducer time history is predominately noise.

FILTERING AND DIFFERENTIATION:

The spherical procedures when compared to the NHTSA or least square procedure is similar to transforming a problem of integrating numeric procedures (low frequency Noise) into a problem of differentiation and algebric numerical procedures (high frequency noise). Therefore, different types of noise affect the various procedures, differently. This effect is greater for the angular velocity than it is for the angular acceleration. In general, for short duration, impacts with significant high-frequency components, the least squares and NHTSA procedure track the angular velocity with a greater accuracy than the spherical method. For long-duration impacts, the spherical method tends to track the angular velocity better than the least squares or NHTSA method. Therefore, it is more important to minimize high frequency noise, such as Gaussian noise, for the spherical method than it is for the least squares, or NHTSA procedure. Methods of statistically reducing Gaussian noise have been presented elsewhere (6). Once these procedures have been applied, differentiation is completed in the frequency domain.

INTRODUCED SIGNALS:

A number of different artificial test signals with introduced noise were evaluated. The artificial signals represent both direct and indirect impacts to a rigid body. These artificial signals were used to compare the three systems. However, for the purpose of this paper, only the comparison of the NHTSA system to the spherical system will be presented. One of the artificial test signals having higher angular velocity than normally seen in an impact environment was used to illustrate the general results for mixed noise. The following comparison of the NHTSA system to the spherical system, illustrates the general difference between the spherical and other systems that obtain angular velocity from integration.

COMPARISON:

Figure 4 illustrates the acceleration of one of the triaxes presented in its instrument frame. This acceleration contains both linear and angular components and represents what might be measured in the laboratory in a test.

Figure 5 gives an example of the exact angular acceleration. In this example, rotation is significant in all three directions.

Figure 6 contains three angular accelerations. These angular accelerations are the most significant component for: 1) the exact angular acceleration, 2) the angular acceleration derived from the NHTSA procedure, and 3) the solution derived from the spherical procedure, in the global instrument frame. This figure illustrates the general results obtained for all of the different test signals. In general, for both procedures, the angular acceleration differs slightly from the true angular acceleration by about the same amount.

Figure 7 contains nine angular velocities. This figure illustrates the general result obtained from both procedures for all of the different test signals. For both procedures, the angular velocity differs from the true angular velocity. However, the spherical procedure tends to be much closer after 75 msec.

Figure 8 contains nine angles in the laboratory frame. This figure illustrates the divergence of the spherical method over time from the exact signals. Because, for the spherical method, the angles are obtained from integration of the angular velocity, error will accumulate. However, as presented in the figure, the error will not accumulate as fast as double integration of the angular acceleration. The implication from this is that when using the spherical method, if the signals are not exact, integration of the angular acceleration does not produce accurate angular velocities over long durations.

This procedure has been used to determine the motion of a vehicle in a barrier crash as well as the motion of the pelvis and thorax of a Hybrid III dummy. However, none of the motions observed in these tests are as complicated as the mathematical simulation above and comparisons have only been made to film data for displacements. Therefore, these data have limited utility in terms of validation, even though they represent a necessary condition. One example is given: For the motion of the thorax of a Hybrid III dummy during an air bag test (Figure 9). The crosses in the figure are the displacements obtained from the film data.

OBSERVATIONS:

1. Angular velocity can be measured directly using linear accelerometers.

2. The potential advantage of the spherical procedure is in the measurement of angular motion. Angular velocity will not diverge or "blow up". Error in the angular velocity of each point in time will not in general accumulate, but represent the noise, or error in the signal at that point in time.

3. Error in the angles of rotation, in the spherical procedure, does not accumulate as fast as in those procedures that derive the angles from double integration.

4. In the spherical technique, when error is part of the acceleration signal, the integration of the angular acceleration does not equal the angular velocity.

5. For short-duration signals, the least squares technique, and the NHTSA technique are more accurate in producing angular velocity. How short the signals need to be for this to be true, depends on the type of noise in the signals (for the test signals used here that duration was 10-20 msec).

ACKNOWLEDGMENTS:

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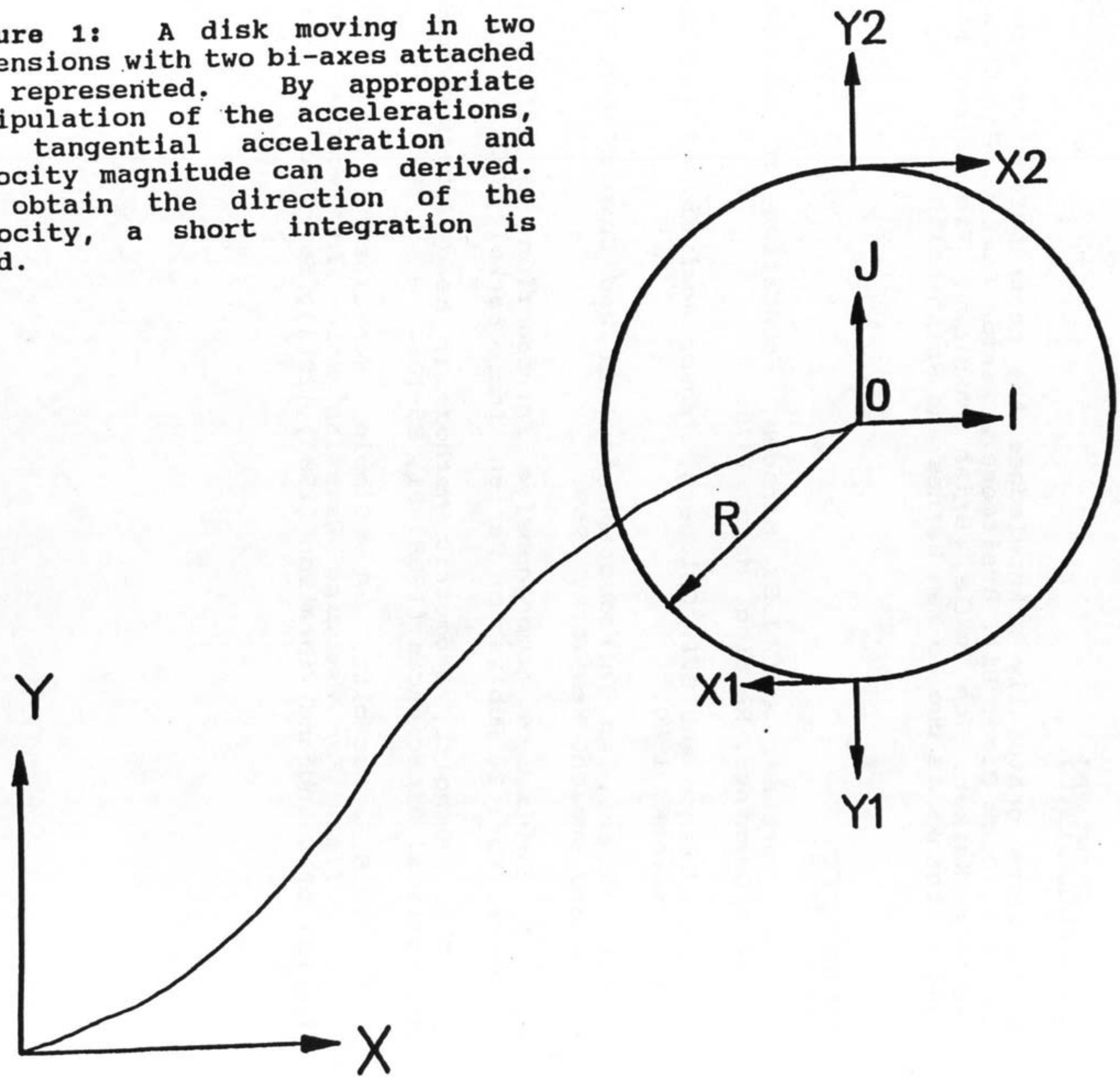


Figure 1: A disk moving in two dimensions with two bi-axes attached is represented. By appropriate manipulation of the accelerations, the tangential acceleration and velocity magnitude can be derived. To obtain the direction of the velocity, a short integration is used.

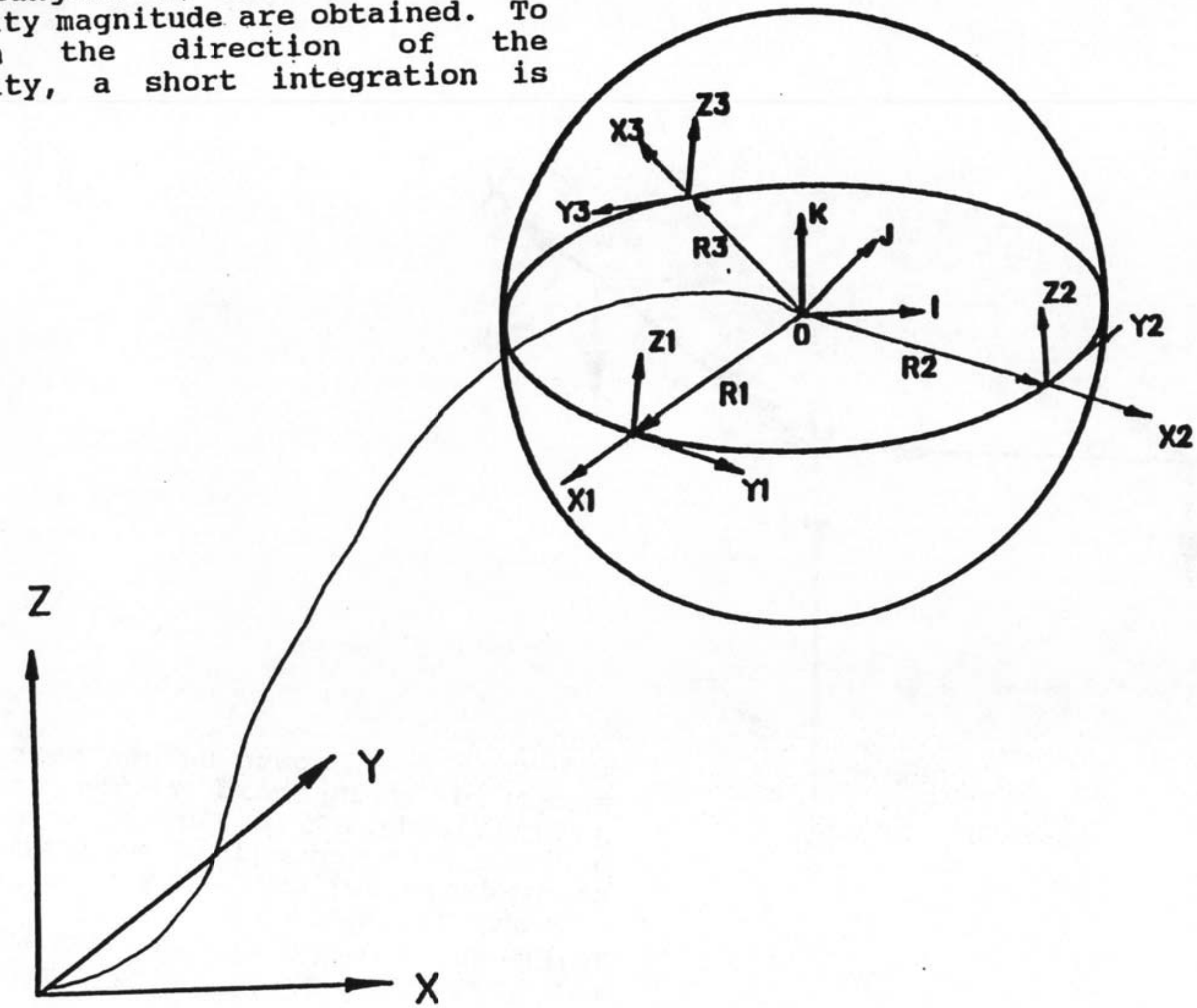


Figure 2: A sphere moving in three dimensions with three triaxes is represented. By appropriate manipulation of the accelerators, the tangential acceleration and velocity magnitude are obtained. To obtain the direction of the velocity, a short integration is used.

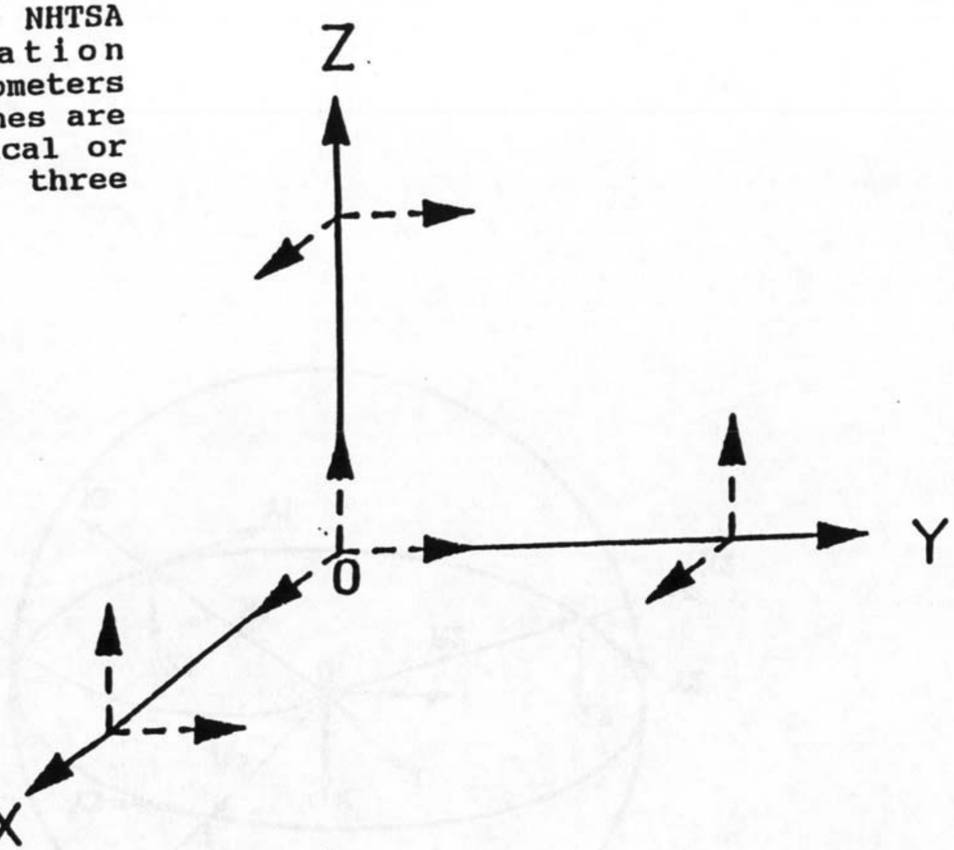
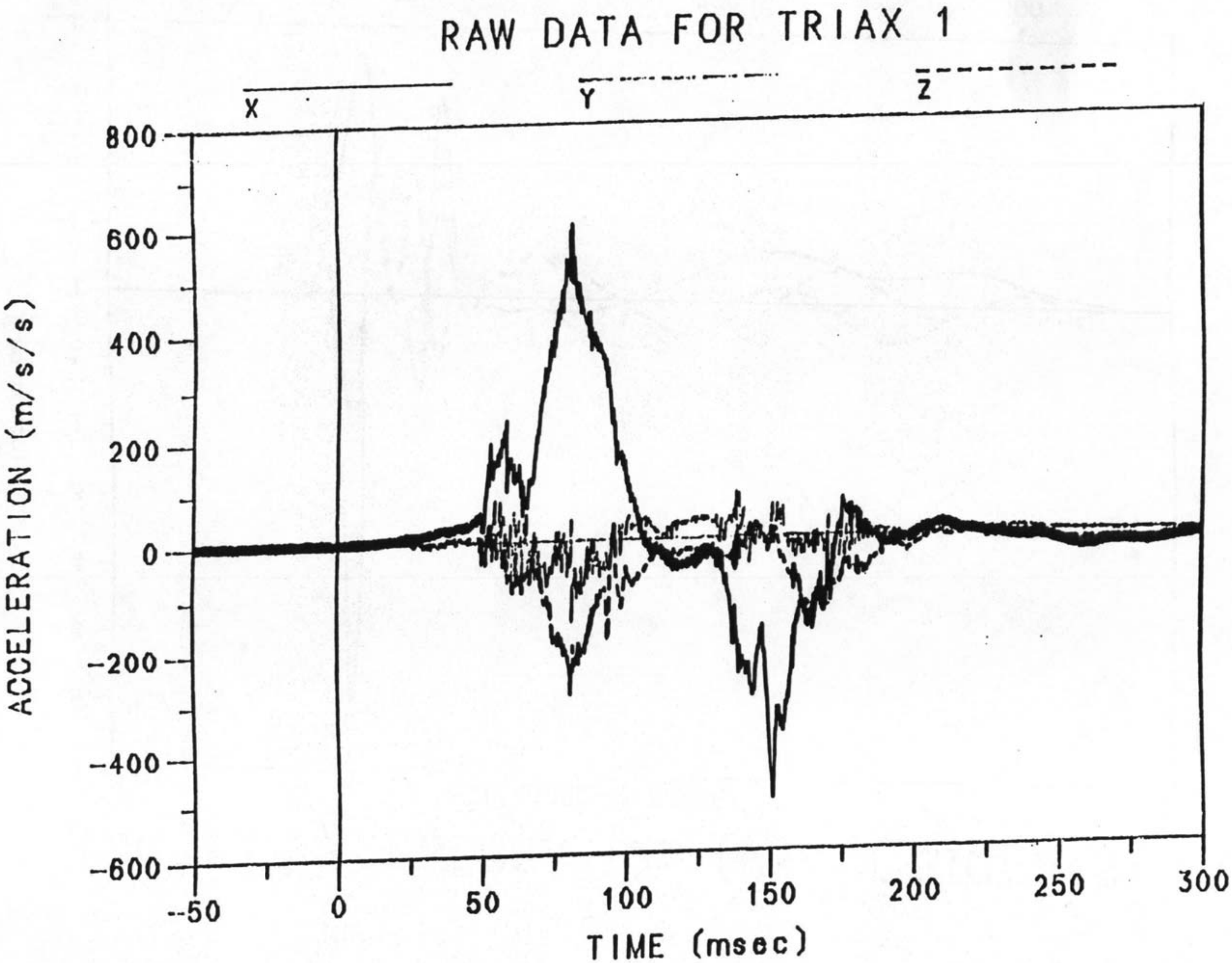


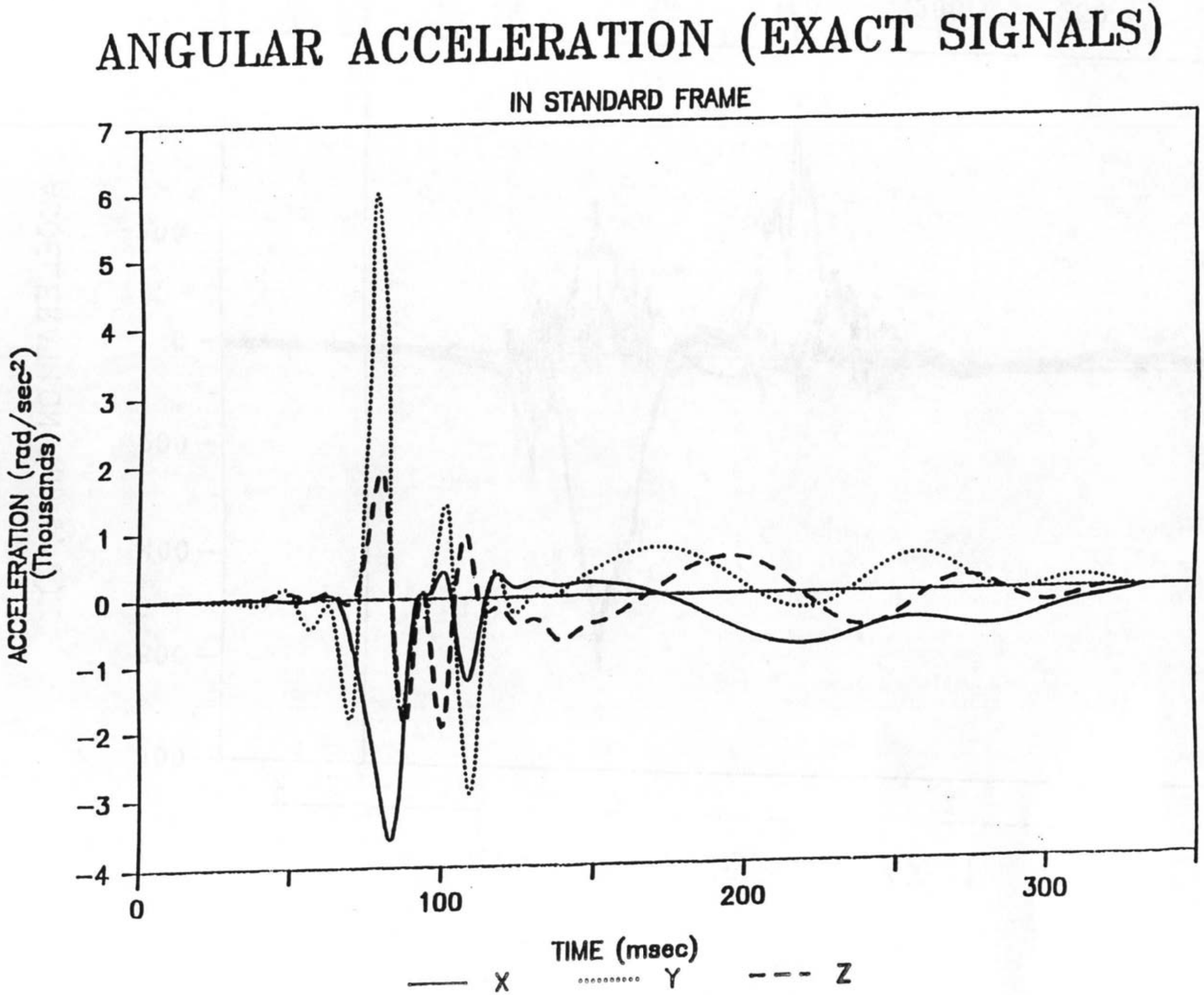
Figure 3: Four triaxes are represented. To obtain the NHTSA (W.S.U.) instrumentation configuration, the accelerometers represented by the dotted lines are used. To obtain the spherical or least square procedure, any three triaxes can be used.

Figure 4:



RAW DATA FOR TRIAX 1
X
Y
Z
ACCELERATION (m/s/s)
800
600
400
200
0
-200
-400
-600
--50
0
50
100
150
200
250
300
TIME (msec)

Figure 5:



ANGULAR ACCELERATION (EXACT SIGNALS)
IN STANDARD FRAME
ACCELERATION (rad/sec²)
(Thousands)
7
6
5
4
3
2
1
0
−1
−2
−3
−4
0
100
200
300
TIME (msec)
X
Y
Z

Figure 6:

COMPARISON OF ANGULAR ACCELERATION

IN STANDARD FRAME

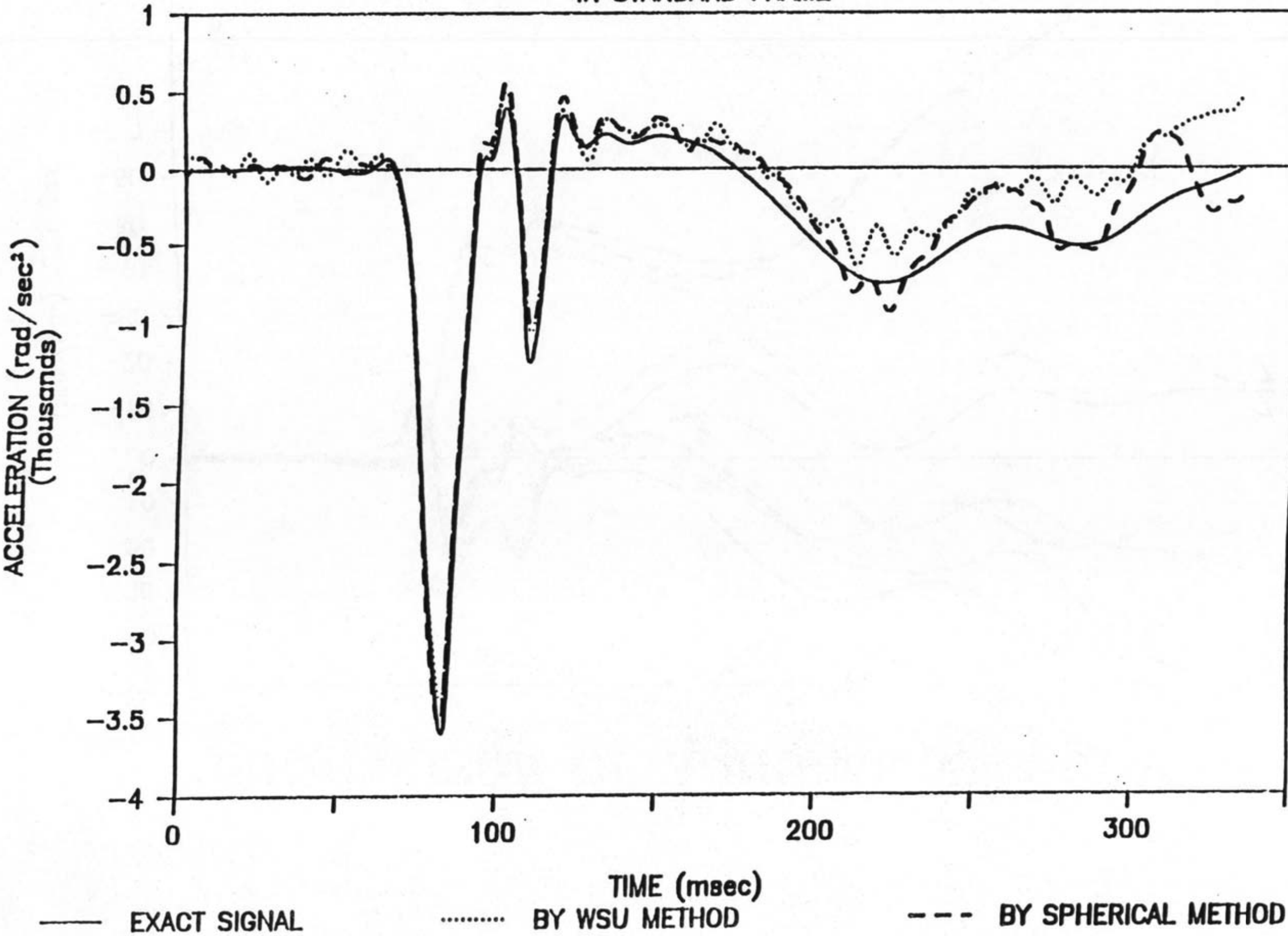
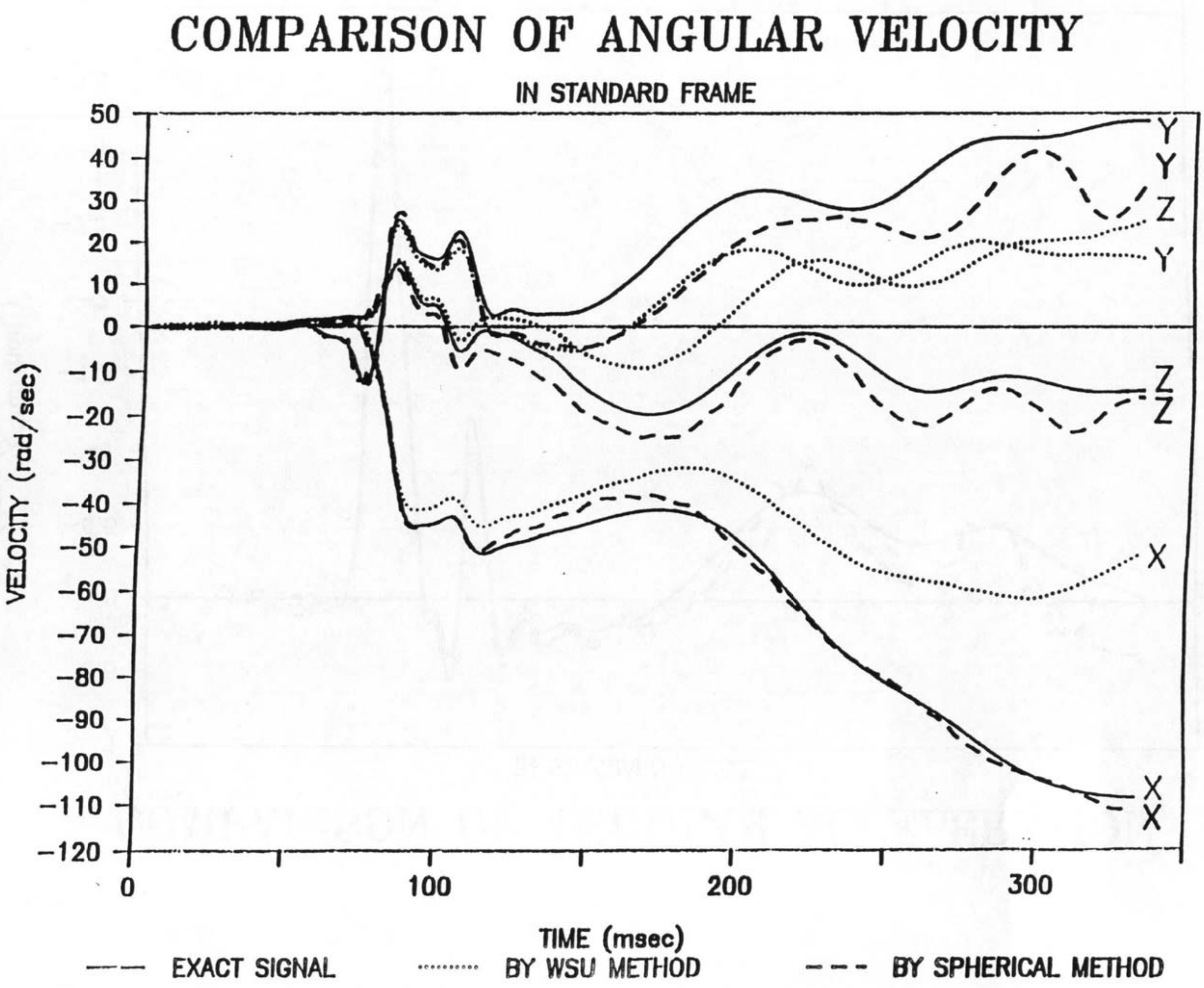
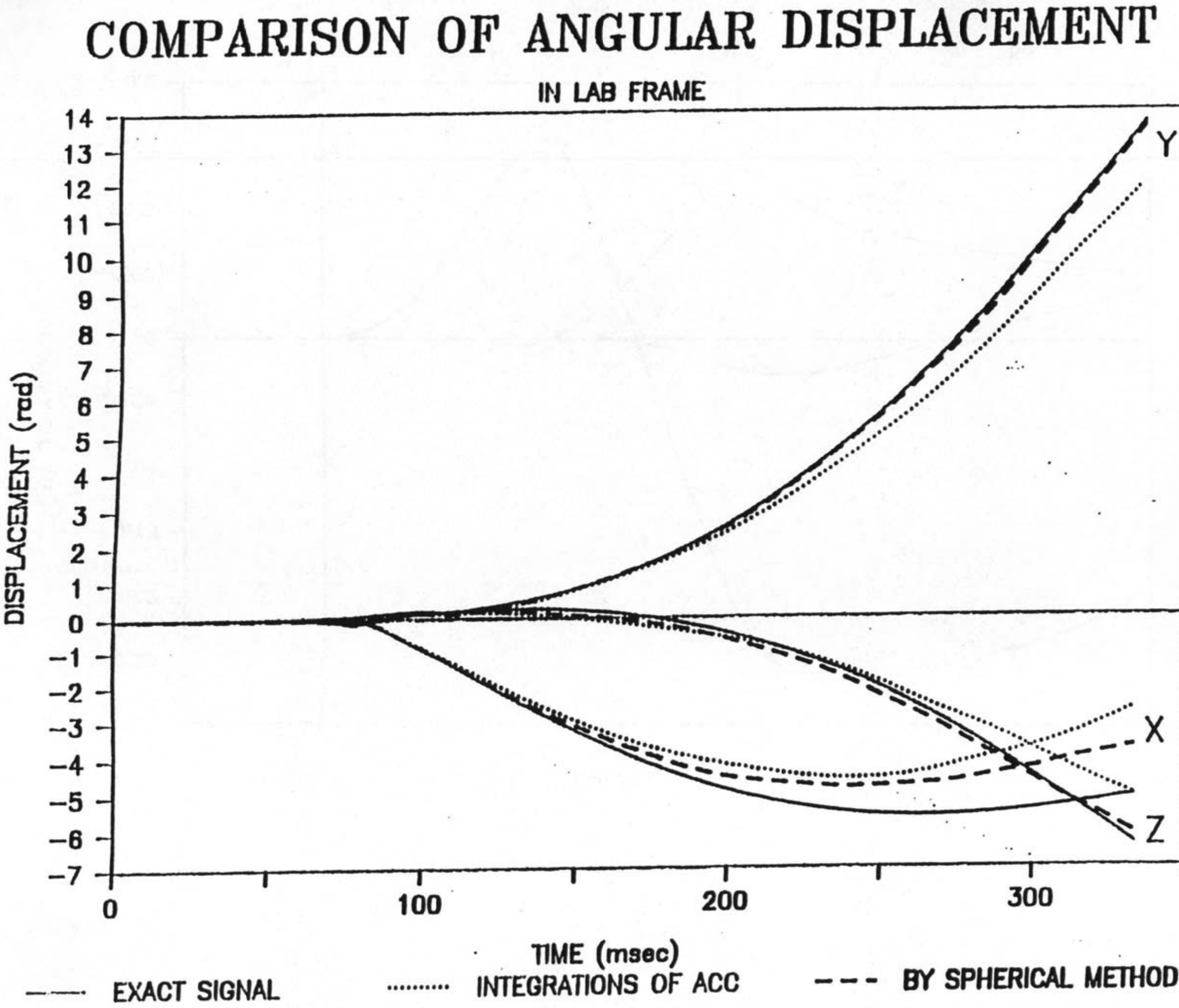


Figure 7:



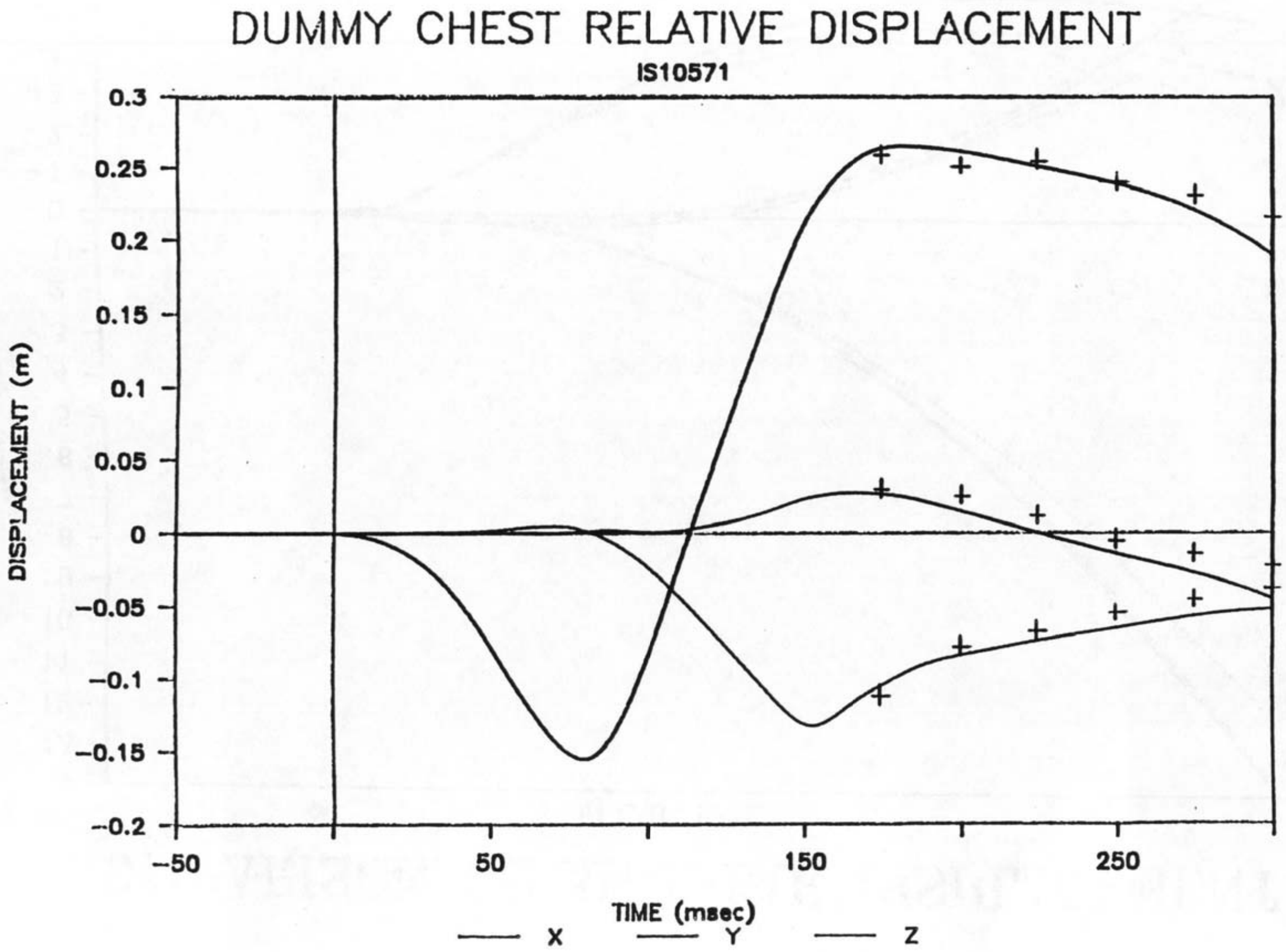
COMPARISON OF ANGULAR VELOCITY
IN STANDARD FRAME
VELOCITY (rad/sec)
50
40
30
20
10
0
−10
−20
−30
−40
−50
−60
−70
−80
−90
−100
−110
−120
0
100
200
300
TIME (msec)
Y
Y
Z
Y
Z
Z
X
X
X
EXACT SIGNAL
BY WSU METHOD
BY SPHERICAL METHOD

Figure 8:



COMPARISON OF ANGULAR DISPLACEMENT
IN LAB FRAME
DISPLACEMENT (rad)
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−5
−6
−7
0
100
200
300
TIME (msec)
Y
X
Z
EXACT SIGNAL
INTEGRATIONS OF ACC
BY SPHERICAL METHOD

Figure 9:


DUMMY CHEST RELATIVE DISPLACEMENT
IS10571
DISPLACEMENT (m)
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-50
50
150
250
TIME (msec)
X
Y
Z

PAPER: Direct Measurement of Angular Velocity

SPEAKER: Guy Nusholtz, Chrysler, C.I.G.

Question: Rolf Eppinger, NHTSA

Could you tell us a little about the physical configuration of your accelerometers inside the dummy head?

Answer: First, it's not in the dummy head. They're in the chest and the pelvis. I can tell you about the proposed configuration in the dummy head if you're interested in that. There's a couple of standard positions in the dummmy head where you can place the accelerometers and what we did was put one in the front, one on the top and one in back on the skull cap and then there's one over to the side in case you want to use the 3-2-2 method. On the dummy thorax what we've done was put one in the standard location and then on the side of the dummy, that steel structure, we just put two triax's, just on the width of the spine box. On the pelvis, we've got one on the standard pelvis location and then we've got two up above that where the soft rubber abdomen sits.